SYNOPSIS OF THE THESIS

ENTITLED

A study of mathematical models through system of fractional differential equations and its solutions

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Ph.D. Synopsis

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1. Introduction

Fractional differential equations are very important in many fields like Fluid Mechanics, Physics, Optics, Electrochemistry of Corrosion, Engineering, Viscoelasticity, Electrical Networks, Biology, and control Theory of Dynamics Systems. We discuss basic definition of fractional differential equation, namely Riemann-Liouvlle, Caputo sense, Grunwald Letnikov, Mittag etc. With the help of definition we emphasize on basic method for solving of linear fractional differential equations by Laplace transform. Fractional differential equation model can be used in biology for investigation of diseases, heat transfer, fluid dynamics, cosmology, signals and control system etc.

In the recent decades the Calculus has played an important role in understanding and development of the various models and phenomenon in science and technology with tremendous progress in Physics, Image processing, Diffusion, Dynamical Processes in self similar and porous structure, Electrochemistry of corrosion, Electric networks, Fluid flow, Computer patterns, Theory of Cosmology, Neural Networks, Biology and many others.

After studding the physical and geometrical approaches of the Fractional calculus, we find the keen interest in the study of the topic through the application aspects. It will make the view of the Fractional calculus wider and fruitful. There are wide ranges of models of system of

equations in the study of heat and mass transfer which can be sincerely modified to a Fractional approach to make the model more effective and able to be solved by using various types techniques.

Fractional calculus has attracted most of the researchers in the field of image processing to enhance the image, contrast the image, edge detection of the image, reduction of noise and in other cases. We are going to extend this concept to analyze the image as well as properties moving objects properties more efficiently by using Fractional calculus.

It has been observed that the various models in the theory of relativity, cosmology and biology can be illustrated and analyze through 'Fractional Differential Equation'. The modified model will define the modern problems in cosmology.

This synopsis 'A study of mathematical models through system of fractional differential equations and its solutions' based on the definitions of fractional derivative and integration, also on special functions, applications and model. We will also approach it to solve the model with the appropriate mathematical technique to get the better results. To get the better understanding, we will use different types of mathematical software like 'Matlab', 'Mathematica', 'Maple', 'Latex' and 'Scilab' etc. as per the requirement of the research work. The figures and numerical results will be handled by this software.

Definition of the Riemann Liouville Fractional Integral:

The Riemann-Liouville fractional integral defined as

$$ {}_aD_x^{-\alpha} f(x) = \frac{1}{\Gamma(\alpha)} \int_a^x (x-t)^{\alpha-1} f(t) dt $$

Where $\alpha > 0$ is any non-negative real number, f(x) be piecewise continuous on (a, x) and integrable on any finite subinterval of [a, x].

Definition of the Riemann Liouville Fractional Derivative:

The fractional derivative can be defined using the definition of the fractional integral. Suppose that on

$n-1 < \alpha \leq n$ and n is smallest integer greater than on α. Then, the fractional derivative of f(x) of order α > 0, is

$$ {}_aD_x^{-\alpha} f(x) = \frac{d^n}{dx^n} [\, {}_aD_x^{-(n-\alpha)} f(x)] $$

$$ = \frac{1}{\Gamma(n-\alpha)} (\frac{d}{dx})^n \int_a^x (x-t)^{n-\alpha-1} f(t) dt $$

Definition of the Caputo's Fractional Derivative:

The Caputo's fractional derivative of f(x) of order α > 0, is

$$ {}_a^C D_x^{-\alpha} f(x) = \frac{1}{\Gamma(n-\alpha)} \int_a^x (x-t)^{n-\alpha-1} f^n(t) dt $$

Where $n-1 < \alpha \leq n$

Origin of the research Problem:

2. Review of Literature and Development in the subject (Previous work done in the related area):

Fractional calculus was born in 1695. The question of whether the meaning of a derivative to an integer order n could be extended to still be valid when n is not an integer. This question was first raised by L'Hospital (1661-1704) in 1695. On that day, in a letter to Leibniz (1646-1716), he posed a question about $\frac{D^n(x)}{Dx^n}$, Leibniz notation for the nth derivative of the linear function f(x) = x. L'Hospital curiously asked what the

result would be if n=1/2. Leibniz responded that it would be an apparent paradox, from which one day useful consequences will be drawn.

Fractional calculus is three centuries as old as the conventional calculus, i.e. a system containing fractional derivatives and integrals. But remarkable work began only in the nineteenth century with many great mathematicians like Euler, P.S. Laplace (1812), J. B. J. Fourier (1822), N. H. Abel (1823-1826), J. Liouville (1832-1873), B. Riemann (1847), Holmgren (1865-67), A.K. Gunwald (1867-1872), A.V. Letnikov (1868-1872), H. Laurent (1884), P.A. Nekrassov (1888) etc., contributing to its development. However, recent attempt is on to have definition of fractional derivative as local operator specifically to fractal science theory. Perhaps, Fractional calculus is the calculus of twenty first century. Heaviside (1922), Buss (1929), Goldman (1949), Starkey (1954), Holbrook (1966), Oldham and Spanier (1974), L. Debnath (1992), Miller and Ross (1993), R.K. Saxena (2002), Igor Podlubny (2003), S. Saha Ray (2005), and several others devoted additionally very reliable contribution to the subject.

In Fluid Dynamics, the role of Fractional calculus has been explained by Abu Arqum (2016) in the field of heat and mass transfer the model can be formulated in the form of Fractional calculus.

A new derivative with non local and no singular kernel is proposed by Caputo sense and it was followed by some related theoretical and applied result. Author used some useful properties of the new derivative are presented and applied to solve the fractional heat transfer model in Abdon and Dumitru (2016).

The Theory of Relativity and Cosmology involves lot of applications of Fractional calculus as well mentioned in El –Nabulsi, R. A. *at. al.* V. K. Schigolev, V. Daftardar-Gejji at. al. The same phenomenon may be applied on different models of cosmology by D. D, Pawar *et. al.*

Pawar et. al. (2011) investigated string cosmology model in presence of massless scalar field in modified theory of general relativity. In this paper the model is constructed for geometric string.

Some cases used integral transform to solve fractional differential equations. Generally integer order derivatives and integrals have clear physical and geometrical interpretation. However, in case of fractional order integration and differentiation, analysis of Fractional differential equation, Fractional diffusion equation, Fractional relaxation equation, which represent a rapidly growing field both in theory and in application to the real world problem Leszczynski at. al. (2011).

D. D. Pawar et al (2018) studied the existence and uniqueness results of solutions for boundary value problems for Hilfer-Hadmard-Type fractional differential equations of his form using fixed point theorem.

Authors compare the performance between systems of ordinary and fractional differential equations involving the Caputo fractional derivative sense depicting the mathematical models of diseases. In order to understand the origins of both approaches as mean-field approximations of integer and fractional stochastic process, fractional linear birth and death processes D. D. Pawar *et. al.* in 2020.

3. Objectives of Research / Proposed Hypothesis:

The main objectives of this study are

I. To study a basic concept of fractional calculus.

II. To study in detail of fractional differential equations.

III. To study existing methods, models and recent applications.

IV. The carried out research work will be published in the well reputed journals.

4. Methodology to be adopted:

I. Initially, for the review purpose relevant research papers, review articles, books, journals etc. will be collected through internet, various institutions and University.

II. To participate in conferences, seminars, webinar, workshops.

III. To obtain an alternative method of fractional differential equation applying new technique and compare our result with obtained already results.

IV. To study mathematical software.

5. Importance of study:

Interdisciplinary Relevance:

The issue has great importance because it is an interdisciplinary work related with the branch of science and technology like Physics, Chemistry, Biology, Civil engineering, Mechanical Engineering and Computer Engineering etc.

There are a number of intriguing questions that remains unanswered about fractional differential equations.

International & National Status:

Lot of work has been carried out by many national and international researchers on fractional differential equation. The works of few amongst them are quoted in the direct and indirect problems.

6. Proposed work Plan / Formulation & Structure of Study:

Year wise plan of work and targets to be achieved are as follows.

1. **First Year:**
   i. To refer the reference books on differential equation and fractional differential equation.
   ii. To collect the references on the proposed research topic.
   iii. To study the collected research material.
   iv. To attend the seminars/conferences/workshops on relevant topic.
   v. To develop new condition on the topic.
2. **Second Year:**
   i. To attend the seminars/conference/workshops on relevant topic.
   ii. To formulate new technique.
   iii. To study existing and new technique.
   iv. To construct mathematical model to related work.
   v. To check whether these models are practically feasible or not.
3. **Third Year:**
   i. To study applications of obtained results.
   ii. To publish the work in reputed journals.
   iii. To submit the work to the University.

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